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Construction of New Variable Separation Excitations via Extended Projective Ricatti Equation Expansion Method in (2 + 1)- Dimensional Dispersive Long Wave Systems

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Utilizing the extended projective Ricatti equation expansion method, abundant variable separation solutions of the (2 + 1)-dimensional dispersive long wave systems are obtained. From the special variable separation solution (38) and by selecting appropriate functions, new types of interaction between the multi-valued and the single-valued solitons, such as semi-foldon and dromion, semi-foldon and peakon, semi-foldon and compacton are found. Meanwhile, we conclude that the solution v is essentially equivalent to the "universal" formula (1).

KEY WORDS: variable separation excitations; extended projective Ricatti equation expansion method; DLW system.

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1. INTRODUCTION

In recent decades, there has been noticeable progress in the study of the soliton theory. Some authors are interested in seeking soliton-like solutions (Wang, 1995; Fan, 2001; Parkes *et al.*, 2002; Conte and Musette, 1992; Yan, 2003; Chen and Li, 2004), because the waveforms can change in different mechanisms and it usually has travelling wave solutions. Others are devoted to finding rich localized coherent structures by now called the multilinear variable separation approach (MLVSA) (Lou and Lu, 1996; Tang *et al.*, 2002; Tang and Lou, 2003a; Zheng and Chen,

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2004; Zheng and Sheng, 2003; Zhang, 2001, 2002; Hong *et al.*, 2003; Ruan and Chen, 2001, 2003).

The MLVSA has been a nearly systematic process to solve $(2+1)$-dimensional nonlinear evolution systems. And via the MLVSA it has been found that a quite "universal" formula

$$U \equiv \frac{\Delta p_x q_y}{(a_0 + a_1 p + a_2 q + a_3 pq)^2}, \quad \Delta \equiv \lambda(a_1 a_2 - a_0 a_3), \tag{1}$$

which is valid for the suitable physical fields or potentials for a large type of $(2+1)$-dimensional physically interesting nonlinear models, including the Davey-Stewartsen (DS) equation, the dispersive long wave (DLW) equation, the Broer-Kaup-Kupershimidt (BKK) system, the Nizhnik-Novikov-Veselov (NVV) equation, and so on (Lou and Lu, 1996; Tang *et al.*, 2002; Tang and Lou, 2003a; Zheng and Chen, 2004; Zheng and Sheng, 2003; Zhang, 2001, 2002; Hong *et al.*, 2003; Ruan and Chen, 2001, 2003). In expression (1), $p \equiv p(x, t)$ is an arbitrary function of $\{x, t\}$, $q \equiv q(y, t)$ may be either an arbitrary function of $\{y, t\}$ or an arbitrary solution of a Riccati equation, while λ, a_0, a_1, a_2 and a_3 are taken as constants. In usual cases, the constant $\lambda = \pm 2$ or $\lambda = \pm 1$. Using the formula (1), the quite rich localized excitations, such as lumps, dromions, peakons, compactons, foldons, ring solitons, fractal solitons, chaotic soliotns and so on (Lou and Lu, 1996; Tang *et al.*, 2002; Tang and Lou, 2003a; Zheng and Chen, 2004; Zheng and Sheng, 2003; Zhang, 2001, 2002; Hong *et al.*, 2003; Ruan and Chen, 2001, 2003) are obtained, and the novel interactive behavior among the same types and various types of soliton excitations are revealed. For example, The interactions between solitons like dromion and dromion, peakon and peakon, compacton and compacton, foldon and foldon have also been studied (Lou and Lu, 1996; Tang *et al.*, 2002; Tang and Lou, 2003a; Zheng and Chen, 2004; Zheng and Sheng, 2003; Zhang, 2001, 2002; Hong *et al.*, 2003; Ruan and Chen, 2001, 2003). More recently, the interactions between dromion and compacton, peakon and compacton, dromion and peakon were reported in Zhang *et al.* (2004). From above mentioned, one can see that the interactions were discussed between either single-valued and single-valued solitons, or multi-valued and multi-valued solitons(foldons). To our knowledge, the interactions between multivalued and single-valued solitons, such as semi-foldon and dromion, semi-foldon and peakon, semi-foldon and compacton, which are focused in the present paper, were little reported in previous literature.

Among these approaches which are usually used to search for travelling wave solutions (Wang, 1995; Fan, 2001; Parkes *et al.*, 2002; Conte and Musette, 1992; Yan, 2003; Chen and Li, 2004) since the first step one may take is travelling reduction, the extended projective Ricatti equation expansion method is an simple and effective method. Now a significant and interesting issue is whether all the localized excitations based on MLVSA can be re-derived by the extended projective Ricatti equation expansion method (Conte and Musette, 1992; Yan, 2003;

Chen and Li, 2004). Another crucial question is whether there exist some similar localized or new localized structures in above mentioned systems. To answer these questions, we take the (2 + 1)-dimensional dispersive long wave equation(DLWE)

$$\begin{aligned} u_{ty} + v_{xx} + (uu_y)_x &= 0, \\ v_t + (uv)_x + u_{xxy} &= 0 \end{aligned} \tag{2}$$

as a concrete example. This equation was introduced by Boiti *et al.* (1987) as a compatibility for a "weak" lax pair. The (1 + 1)-dimensional DLWE ($y = x$ of Eq. (2)) is called the classical Boussinesq equation. There exist a large number of papers to discuss the possible applications and exact solutions of the (1 + 1)-dimensional DLWE (Musette and Conte, 1994) Various interesting properties of the (2 + 1)-dimensional DLWE have been studied by many authors (Paquin and Winternitz, 1990; Lou, 1993, 1994, 1995; Tang and Lou, 2003b). For example, In Paquin and Winternitz (1990), showed that the symmetry algebra of Eq. (1) is infinite-dimensional and Kac-Moody-Virasoro structure. In Lou (1995), outlined nine types of two dimensional similarity reductions. In Tang and Lou (2003b), folded solitary waves and foldons was defined and studied both analytically and graphically.

In our present paper, we obtain some solutions of Eq. (2) with certain arbitrary functions by the extended projective Ricatti equation expansion method. Based on these solutions and by selecting appropriate functions, new types of interaction between the multi-valued and the single-valued solitons, such as semi-foldon and dromion, semi-foldon and peakon, semi-foldon and compacton are presented.

2. THE GENERALIZED PROJECTED RICATTI EQUATION EXPANSION METHOD

Consider a given nonlinear evolution equation with independent variables $x = (x_0 = t, x_1, x_2, \ldots, x_m)$ and dependent variable u

$$F\big(u, u_t, u_{x_i}, u_{x_i x_j}, \ldots\big) = 0. \tag{3}$$

Step 1 We assume that (3) has the following formal solutions:

$$u(x) = a_0(x) + \sum_{i=1}^{l} f^{i-1}(\omega(x))[a_i(x) f(\omega(x)) + b_i(x) g(\omega(x))], \tag{4}$$

with

$$f' = \epsilon f g, \quad g' = R + \epsilon g^2 - r f, \quad \epsilon = \pm 1, \tag{5}$$

$$g^2 = -\epsilon\Big[R - 2rf + \frac{r^2 + \mu}{R} f^2\Big], \quad R \neq 0, \quad \mu = \pm 1, \tag{6}$$

where $x = (x_0 = t, x_1, x_2, \ldots, x_m)$, R, r are constants and' denotes $\frac{d}{d\omega}$. It is easy to see that Eq. (5) and (6) admits the following solutions:

Case 1 $\epsilon = -1, \mu = -1$

$$f_1 = \frac{R\mathrm{sech}(\sqrt{R}\omega)}{r\mathrm{sech}(\sqrt{R}\omega) + 1}, \quad g_1 = \frac{\sqrt{R}\tanh(\sqrt{R}\omega)}{r\mathrm{sech}(\sqrt{R}\omega) + 1}. \tag{7}$$

$$f_2 = \frac{4R\mathrm{sech}(\sqrt{R}\omega)}{4r\mathrm{sech}(\sqrt{R}\omega) + 3\tanh(\sqrt{R}\omega) + 5},$$

$$g_2 = \frac{\sqrt{R}(5\tanh(\sqrt{R}\omega) + 3)}{4r\mathrm{sech}(\sqrt{R}\omega) + 3\tanh(\sqrt{R}\omega) + 5}. \tag{8}$$

Case 2 $\epsilon = -1, \mu = 1$

$$f_3 = \frac{R\mathrm{csch}(\sqrt{R}\omega)}{r\mathrm{csch}(\sqrt{R}\omega) + 1}, \quad g_3 = \frac{\sqrt{R}\coth(\sqrt{R}\omega)}{r\mathrm{csch}(\sqrt{R}\omega) + 1}. \tag{9}$$

Case 3 $\epsilon = 1, \mu = -1$

$$f_4 = \frac{R\sec(\sqrt{R}\omega)}{r\sec(\sqrt{R}\omega) + 1}, \quad g_4 = \frac{\sqrt{R}\tan(\sqrt{R}\omega)}{r\sec(\sqrt{R}\omega) + 1}. \tag{10}$$

$$f_5 = \frac{\sqrt{R}\csc(\sqrt{R}\omega)}{r\csc(\sqrt{R}\omega) + 1}, \quad g_5 = -\frac{\sqrt{R}\cot(\sqrt{R}\omega)}{r\csc(\sqrt{R}\omega) + 1}. \tag{11}$$

Step 2 Determine the parameter l by balancing the highest order derivative terms with the nonlinear terms in Eq. (3).

Step 3 Substituting (4) with (5) and (6) into (3) yields a set of algebraic polynomials for $f^i g^j (i = 0, 1, \ldots; j = 0, 1)$. Eliminating all the coefficients of the powers of $f^i g^j$, yields a series of algebraic equations, from which the parameters $a_0, a_i, b_i (i = 1, \ldots, l)$ and ω are explicitly determined.

Step 4 Substituting a_0, a_i, b_i, ω obtained in Step 3 into (4) with (7)–(11), we can then derive the solutions of (3).

3. NEW VARIABLE SEPARATION SOLUTIONS OF THE (2 + 1)- DIMENSIONAL DISPERSIVE LONG WAVE EQUATION

To solve the (2 + 1)-dimensional dispersive long wave equation, we consider the following Painlevé-Bäcklund transformation for u and v in (2):

$$u = \pm 2\ln(f)_x + u_0, \quad v = 2\ln(f)_{xy} + v_0, \tag{12}$$

which can be derived from the standard Painlevé truncated expansion, where the functions $u_0 = u_0(x, t)$ and $v_0 = 0$ are seed solutions of (2). Based on (12) and seed solutions, we can straightly obtain a simple relation for u and

$$v = \pm u_y. \tag{13}$$

Inserting (13) into (2) yields

$$\partial_y(u_t \pm u_{xx} + uu_x) = 0. \tag{14}$$

For simplicity and convenience discussions, here we take

$$u_t \pm u_{xx} + uu_x = 0. \tag{15}$$

Now we apply the generalized projected Ricatti equation expansion method to Eq. (14). By balancing the highest order derivative terms with the nonlinear terms in Eq. (14), the ansatz (4) becomes

$$u = a_0(x, y, t) + a_1(x, y, t) f(\omega(x, y, t)) + b_1(x, y, t) g(\omega(x, y, t)), \tag{16}$$

where $a_0(x, y, t), a_1(x, y, t), b_1(x, y, t), \omega(x, y, t)$ are arbitrary functions of $\{x, y, t\}$ to be determined later. Substituting (16) with (5) and (6) into (15), and eliminating the coefficients of the powers of $f^i g^j (i = 0 \sim 4; j = 0, 1)$, one have

$$\begin{aligned} &-3Ra_1{}^2\epsilon w_y w_x(\mu + r^2) \pm 12b_1\epsilon w_x{}^2 w_y r^2\mu + 6b_1{}^2 w_y r^2 w_x\mu \\ &\pm 6b_1\epsilon w_x{}^2 w_y(\mu^2 + r^4) + 3b_1{}^2 w_y w_x(\mu^2 + r^4) = 0, \end{aligned} \tag{17}$$

$$-6a_1\epsilon w_y R b_1 w_x(\mu + r^2) \mp 6a_1 w_y w_x{}^2 R(\mu + r^2) = 0, \tag{18}$$

$$\begin{aligned} &-2Ra_1 b_{1x} w_y(\mu + r^2) \mp 2a_{1y}\epsilon w_x{}^2 R(\mu + r^2) \mp 4a_{1_x}\epsilon w_y w_x R(\mu + r^2) \\ &\mp 4a_1\epsilon w_x w_{xy} R(\mu + r^2) - 2a_1\epsilon w_y w_t R(\mu + r^2) \\ &-2Ra_0 a_1\epsilon w_y w_x(\mu + r^2) - 2Ra_1 b_{1y} w_x(\mu + r^2) - 2Ra_1 b_1 w_{xy}(\mu + r^2) \\ &\mp 12b_1\epsilon w_x{}^2 w_y r R(\mu + r^2) - 2b_1 w_y a_{1_x} R(\mu + r^2) \\ &-7b_1{}^2 w_y w_x R r(\mu + r^2) - 2a_{1y} R b_1 w_x(\mu + r^2) \mp 2a_1\epsilon w_y w_{xx} R(\mu + r^2) \\ &+5a_1{}^2\epsilon w_y w_x r R^2 = 0, \end{aligned} \tag{19}$$

$$\begin{aligned} &\mp 2b_{1y}\epsilon w_x{}^2 R(\mu + r^2) \mp 4b_1\epsilon w_x w_{xy} R(\mu + r^2) \\ &\mp 2b_1\epsilon w_y w_{xx} R(\mu + r^2) - 2b_{1y} R b_1 w_x(\mu + r^2) - Rb_1{}^2 w_{xy}(\mu + r^2) \\ &+a_1{}^2\epsilon w_{xy} R^2 + 6a_1\epsilon w_y R^2 b_1 w_x r \pm 6a_1 w_y w_x{}^2 R^2 r \\ &-2b_1\epsilon w_y w_t R(\mu + r^2) \mp 4b_{1x}\epsilon w_y w_x R(\mu + r^2) \\ &+2a_{1y} R^2 a_1\epsilon w_x - 2b_1 w_y b_{1x} R(\mu + r^2) \\ &-2Ra_0 b_1\epsilon w_y w_x(\mu + r^2) + 2_{a_1}\epsilon w_y R^2 a_{1x} = 0, \end{aligned} \tag{20}$$

$$
\begin{aligned}
&-2{a_1}^2\epsilon w_y w_x R^3 \pm 6a_{1x}\epsilon w_y w_x R^2 r + a_1 a_{1xy} R^2 + a_{1y} R^2 a_{1x} \\
&\quad \pm 7b_1\epsilon {w_x}^2 w_y r^2 R^2 + 3a_1 b_{1y} w_x r R^2 \\
&\quad + 5{b_1}^2 w_y r^2 R^2 w_x \pm 6a_1\epsilon w_x w_{xy} R^2 r + 3a_{1y} R^2 b_1 w_x r \\
&\quad \pm 3a_{1y}\epsilon {w_x}^2 R^2 r + 2{b_1}^2 w_y R^2 w_x \mu \mp b_{1y} w_{xx} R(\mu + r^2) \\
&\quad \mp b_{1xx} w_y R(\mu + r^2) - b_{1y} w_t R(\mu + r^2) - b_1 w_{ty} R(\mu + r^2) \\
&\quad \mp b_1 w_{xxy} R(\mu + r^2) - b_{1t} w_y R(\mu + r^2) \mp 2b_{1xy} w_x R(\mu + r^2) \\
&\quad \mp 2b_{1x} w_{xy} R(\mu + r^2) - b_1\epsilon b_{1xy} R(\mu + r^2) - b_{1y}\epsilon R b_{1x}(\mu + r^2) \\
&\quad - a_{0y} R b_1 w_x(\mu + r^2) - R a_0 b_{1x} w_y(\mu + r^2) \\
&\quad - b_1 w_y a_{0x} R(\mu + r^2) - R a_0 b_{1y} w_x(\mu + r^2) - R a_0 b_1 w_{xy}(\mu + r^2) \\
&\quad \pm 4b_1\epsilon {w_x}^2 w_y R^2 \mu + 3b_1 w_y r R^2 a_{1x} \\
&\quad + 3a_0 a_1\epsilon w_y w_x r R^2 + 3a_1 b_1 w_{xy} r R^2 + 3a_1 b_{1x} w_y r R^2 \\
&\quad \pm 3a_1\epsilon w_y w_{xx} R^2 r + 3a_1\epsilon w_y w_t R^2 r = 0, \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
&2b_1 w_y r R^2 b_{1x} + a_{1y} R^2 b_{1x} + b_1 a_{1xy} R^2 + a_1 b_{1xy} R^2 + b_{1y} R^2 a_{1x} \\
&\quad \pm a_{1y}\epsilon w_t R^2 \pm a_{1xx}\epsilon w_y R^2 \pm a_1\epsilon w_{xxy} R^2 \\
&\quad \mp a_1 w_y {w_x}^2 R^3 + {b_1}^2 w_{xy} r R^2 \pm a_{1y}\epsilon w_{xx} R^2 + a_1\epsilon w_{ty} R^2 + a_{1t}\epsilon w_y R^2 \\
&\quad \pm 2a_{1xy}\epsilon w_x R^2 \pm 2a_{1x}\epsilon w_{xy} R^2 + a_0 a_{1x}\epsilon w_y R^2 \\
&\quad + a_0 a_1\epsilon w_{xy} R^2 + a_0 a_{1y}\epsilon w_x R^2 + a_0 b_1\epsilon w_y w_x r R^2 \pm b_{1y}\epsilon {w_x}^2 R^2 r \\
&\quad + a_1 \in w_y R^2 a_{0x} + a_{0y} R^2 a_1\epsilon w_x \pm b_1\epsilon w_y w_{xx} R^2 r \\
&\quad - a_1\epsilon w_y R^3 b_1 w_x + b_1\epsilon w_y w_t R^2 r + 2b_{1y} R^2 b_1 w_x r \pm 2b_{1x}\epsilon w_y w_x R^2 r \\
&\quad \pm 2b_1\epsilon w_x w_{xy} R^2 r = 0, \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
&a_{0y} R^2 a_{1x} + a_1 a_{0xy} R^2 + a_0 a_{1xy} R^2 + a_{1y} R^2 a_{0x} + 2b_{1y}\epsilon R^2 b_{1x} r \\
&\quad \mp 2a_1\epsilon w_x w_{xy} R^3 \mp 2a_{1x}\epsilon w_y w_x R^3 \\
&\quad + a_{1ty} R^2 + 2b_1\epsilon b_{1xy} R^2 r \pm b_{1xx} w_y R^2 r + b_{1y} w_t R^2 r \pm b_1 w_{xxy} R^2 r \\
&\quad + b_1 w_{ty} R^2 r + b_{1t} w_y R^2 r - a_{1y} R^3 b_1 w_x \\
&\quad - a_1 b_{1y} w_x R^3 - a_1 b_{1x} w_y R^3 - b_1 w_y R^3 a_{1x} \mp a_{1y}\epsilon {w_x}^2 R^3 \\
&\quad \pm b_{1y} w_{xx} R^2 r - a_1 b_1 w_{xy} R^3 \pm 2b_{1xy} w_x R^2 r \pm 2b_{1x} w_{xy} R^2 r \\
&\quad \pm a_{1xxy} R^2 + a_0 b_{1x} w_y r R^2 + a_{0y} R^2 b_1 w_x r - a_1\epsilon w_y w_t R^3 \mp a_1\epsilon w_y w_{xx} R^3
\end{aligned}
$$

$$\mp b_1 \epsilon w_x{}^2 w_y R^3 r - b_1{}^2 w_y R^3 w_x r$$
$$+ b_1 w_y r R^2 a_{0x} + a_0 b_{1y} w_x r R^2 + a_0 b_1 w_{xy} r R^2 - a_0 a_1 \epsilon w_y w_x R^3 = 0, \quad (23)$$
$$a_{0y} R^2 a_{0x} + a_0 a_{0xy} R^2 + a_{0ty} R^2 - b_1 \epsilon b_{1xy} R^3$$
$$- b_{1y} \epsilon R^3 b_{1x} \pm a_{0xxy} R^2 = 0, \quad (24)$$
$$a_{0y} R^2 b_{1x} + b_{1ty} R^2 \pm b_{1xxy} R^2 + b_1 a_{0xy} R^2 + b_{1y} R^2 a_{0x} + a_0 b_{1xy} R^2 = 0. \quad (25)$$

According to (17)–(25) and through careful calculations, we can derive many families of solutions. For the limit of length, we only list two classes of solutions.

$$a_0 = -\frac{\omega_t \pm \omega_{xx}}{\omega_x}, \quad a_1 = \pm \frac{\sqrt{-R\epsilon(\mu + r^2)}\omega_x}{R},$$
$$b_1 = \mp \epsilon \omega_x, \quad \omega = \chi(x,t) + \varphi(y), \quad (26)$$

where $R > 0$ and r are two arbitrary constants. And

$$r = 0, \quad a_1 = 0, \quad a_0 = -\frac{\omega_t \pm \omega_{xx}}{\omega_x}, \quad b_1 = \mp 2\epsilon \omega_x, \quad \omega = \chi(x,t) + \varphi(y). \quad (27)$$

From (26), (27) with (7)–(11), one can get two families of solutions. For simplicity, we only give these solutions from (27) with (7)–(11):

$\epsilon = -1, \mu = -1$

$$u_1 = -\frac{\chi_t \pm \chi_{xx}}{\chi_x} \pm 2\sqrt{R}\chi_x \tan(\sqrt{R}(\chi + \varphi)), \quad (28)$$
$$v_1 = \pm u_{1y} = 2R\chi_x \varphi_y \mathrm{sech}^2(\sqrt{R}(\chi + \varphi)), \quad (29)$$
$$u_2 = -\frac{\chi_{xx} \pm \chi_t}{\chi_x} \pm 2\sqrt{R}\chi_x \frac{5\tanh(\sqrt{R}(\chi + \varphi)) + 3}{5 + 3\tanh(\sqrt{R}(\chi + \varphi))}, \quad (30)$$
$$v_2 = \pm u_{2y} = \frac{32R\chi_x \varphi_y \mathrm{sech}^2(\sqrt{R}(\chi + \varphi))}{[5 + 3\tanh(\sqrt{R}(\chi + \varphi))]^2}, \quad (31)$$

with two arbitrary functions $\chi(x,t)$, $\varphi(y)$ and an arbitrary constant $R(> 0)$.

$\epsilon = -1, \mu = 1$

$$u_3 = -\frac{\chi_t \pm \chi_{xx}}{\chi_x} \pm 2\sqrt{R}\chi_x \coth(\sqrt{R}(\chi + \varphi)), \quad (32)$$
$$v_3 = \pm u_{3y} = -2R\chi_x \varphi_y \mathrm{csch}^2(\sqrt{R}(\chi + \varphi)), \quad (33)$$

with two arbitrary functions $\chi(x,t)$, $\varphi(y)$ and an arbitrary constant $R(> 0)$.

$\epsilon = 1, \mu = -1$

$$u_4 = -\frac{\chi_t \pm \chi_{xx}}{\chi_x} \mp 2\sqrt{R}\chi_x \tan(\sqrt{R}(\chi + \varphi)), \tag{34}$$

$$v_4 = \pm u_{4y} = -2R\chi_x\varphi_y \sec^2(\sqrt{R}(\chi + \varphi)), \tag{35}$$

$$u_5 = -\frac{\chi_t \pm \chi_{xx}}{\chi_x} \pm 2\sqrt{R}\chi_x \cot(\sqrt{R}(\chi + \varphi)), \tag{36}$$

$$v_5 = \pm u_{5y} = -2R\chi_x\varphi_y \csc^2(\sqrt{R}(\chi + \varphi)), \tag{37}$$

with two arbitrary functions $\chi(x, t)$, $\varphi(y)$ and an arbitrary constant $R(> 0)$.

It is noted that the sign "±" in (17)–(37) is matched as following: the above and above signs are composed of one pair, the nether and nether signs constitute another pair. Take example for (28), i.e.

$$u_1 = -\frac{\chi_t - \chi_{xx}}{\chi_x} + 2\sqrt{R}\chi_x \tanh(\sqrt{R}(\chi + \varphi))$$

or

$$u_1 = -\frac{\chi_t - \chi_{xx}}{\chi_x} - 2\sqrt{R}\chi_x \tanh(\sqrt{R}(\chi + \varphi))$$

Remark When $\chi(x, t) = \xi(x) + \tau(t)$, these solutions (28), (29) and (32)–(37) in our paper can degenerate to the solutions (25)–(32) in Zheng *et al.* (2004a), where non-propagating (2 + 1)-dimensional solitons were discussed in detail.

4. NEW TYPES OF INTERACTION BETWEEN SOLITONS IN THE (2 + 1)-DIMENSIONAL DLWE

Since some arbitrariness of the functions $\chi(x, t)$ and $\varphi(y)$ included in above cases, the physical quantities u and v may possess abundant structures. For example, when $\chi(x, t) - \xi(x) + \tau(t)$ and selecting appropriate functions, the rich non-propagating (2 + 1)-dimensional solitons can be found (Zheng *et al.*, 2004b). If $\chi(x, t) - f(kx + ct)$ and $\varphi(y) = g(y)$, then all the solutions of the above cases may show rich localized and propagating excitations. Moreover, one of the simplest travelling wave excitations can be easily obtained by selecting $\chi(x, t) = kx + ct$ and $\varphi(y) = ly$, where k, l, c are arbitrary constants. The localized excitations, such as dromion, peakon, foldon *et al.*, and the interactive behaviors between either single-valued and single-valued solitons, or multi-valued and multi-valued solitons(foldons) in the (2 + 1)-dimensional systems have been discussed (Lou and Lu, 1996; Tang *et al.*, 2002; Tang and Lou, 2003a,b; Zheng and Chen, 2004; Zheng and Sheng, 2003; Zhang, 2001, 2002; Hong *et al.*, 2003; Ruan and Chen, 2001, 2003; Zhang *et al.*, 2004; Lou, 1995). Obviously, these localized excitations above mentioned can be easily recovered by selecting appropriate functions of the expression (38). Here we are interested in revealing some new types of interaction between solitons in the $(2+1)$-dimensional systems such as the interactive behaviors between semi-foldon and dromion, semi-foldon and peakon, semi-foldon and compacton. For simplification in the following discussion, we merely analyze these interaction based on the solutions (29) and rewrite in a simple form (as $R = 1$), that is

$$v \equiv v_1 = 2\chi_x \varphi_y \text{sech}^2(\chi + \varphi). \tag{38}$$

4.1. Interaction Between Semi-Foldon and Peakon

Due to the arbitrariness of the functions in solutions (38), we can find the interaction between semi-foldon and peakon by selecting an arbitrary multi-valued function and an arbitrary single-valued piecewise smooth function, i.e.

$$\chi_x = \text{sech}^2(\zeta) + 0.5\text{sech}^2(\zeta - 0.3t), \quad x = \zeta - 0.5\tanh(\zeta - 0.3t),$$

$$\chi = \int^{\zeta} \chi_x x_\zeta d\zeta, \tag{39}$$

$$\varphi = \begin{cases} \exp(y) & y < 0 \\ -\exp(-y) + 2 & y \geq 0 \end{cases} \tag{40}$$

From Fig. 1, we can see the interaction between multi-valued semi-foldon and single-valued peakon is completely elastic which is similar to the interaction between either single-valued and single-valued solitons (Lou and Lu, 1996; Tang *et al.*, 2002; Tang and Lou, 2003a; Zheng and Chen, 2004; Zheng and Sheng, 2003; Zhang, 2001, 2002; Hong *et al.*, 2003; Ruan and Chen, 2001, 2003; Zhang *et al.*, 2004), or multi-valued and multi-valued solitons (foldons) (Tang and Lou, 2003b) since the wave shapes, amplitudes and velocities of the static peakon and moving semi-foldon are hardly changed after collision. Moreover, the phase shift of the static peakon can be observed. Before the interaction, the static peakon is located at $x = -1.5$ and after the interaction, it is shifted to $x = 1.5$. The phase shift of the static peakon is 3.

4.2. Interaction Between Semi-Foldon and Compacton

Similarly, one of the simplest selections for discussing the interaction between semi-foldon and compacton is to take an arbitrary multi-valued function and an arbitrary single-valued function, i.e.

$$\chi_x = \text{sech}^2(\zeta) + 0.5\text{sech}^2(\zeta - 0.3t),$$

$$x = \zeta - 1.5\ \tanh(\zeta - 0.3t), \quad \chi = \int^{\varsigma} \chi_x x_\varsigma d\zeta, \tag{41}$$

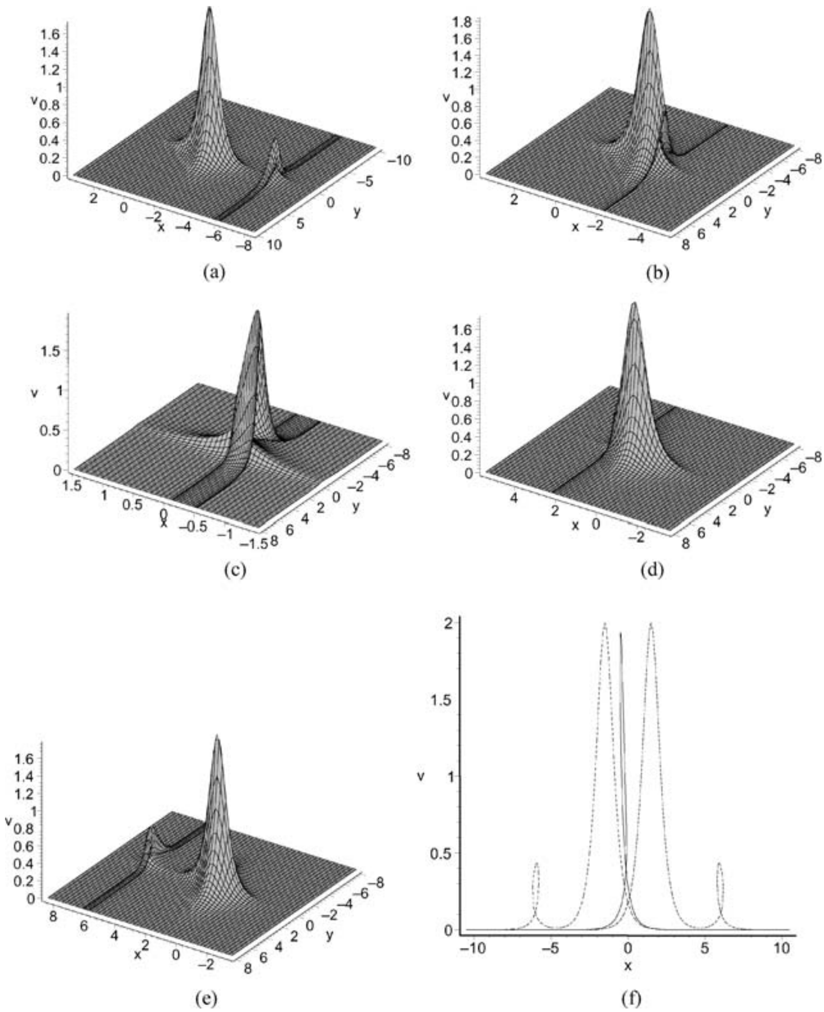

Fig. 1. Evolution profiles of the elastic interaction between semi-foldon and peakon determined by Eq. (38) with Eqs. (39) and (40) at (a) $t = -20$, (b) $t = -7$, (c) $t = -1$, (d) $t = 7$, (e) $t = 20$. (f) The corresponding sectional view at $\{t = -20,\ y = 0\}$ (dotted line before collision), $\{t = -1,\ y = 0\}$(solid line in collision), $\{t = 20,\ y = 0\}$ (dashed line after collision), respectively.

$$\varphi = \begin{cases} 0 & y \le -\frac{\pi}{2} \\ \sin(y) + 1 & -\frac{\pi}{2} < y \le \frac{\pi}{2} \\ 2 & y > \frac{\pi}{2} \end{cases} \tag{42}$$

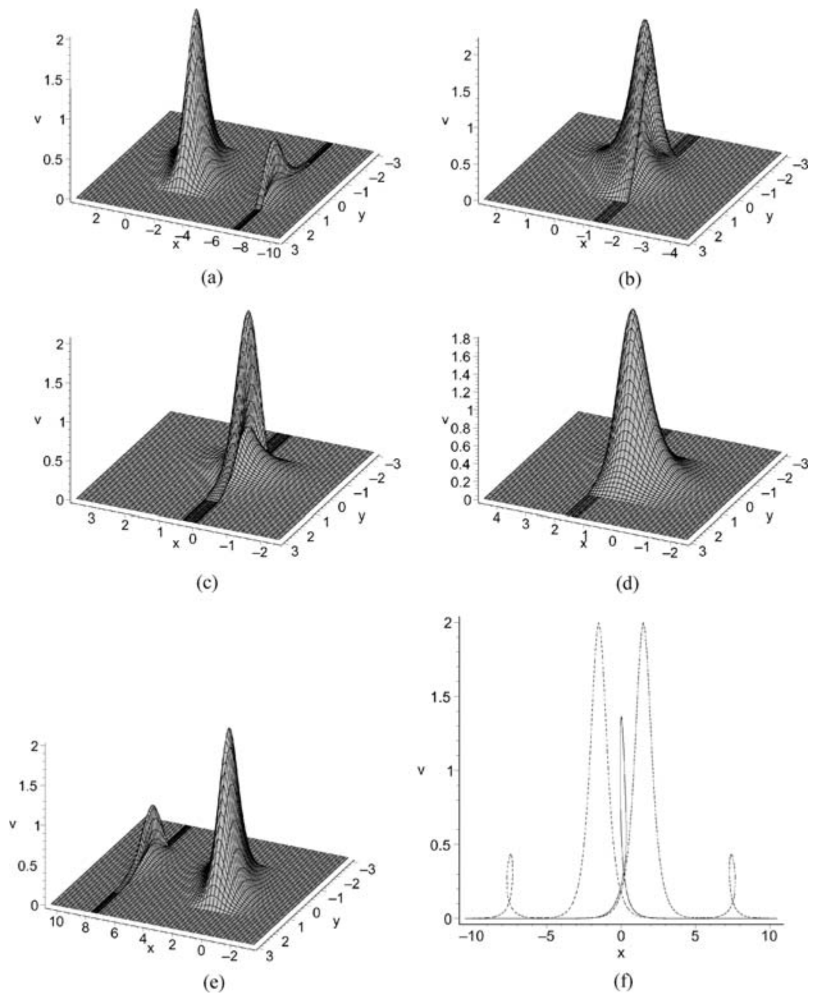

Fig. 2. Evolution profiles of the elastic interaction between semi-foldon and compacton determined by Eq. (38) with Eqs. (41) and (42) at (a) $t = -25$, (b) $t = -5$, (c) $t = 0.5$, (d) $t = 5$, (e) $t = 25$. (f) The corresponding sectional view at $\{t = -25, y = 0\}$ (dotted line before collision), $\{t = 0.5, y = 0\}$ (solid line in collision), $\{t = 25, y = 0\}$ (dashed line after collision), respectively.

From Fig. 2, we can see the interaction between multi-valued semi-foldon and single-valued compacton is also completely elastic which is similar to the interactive behaviors between multi-valued semi-foldon and single-valued peakon. The wave shapes, amplitudes and velocities of the static compacton and moving semi-foldon are nearly unchanged after collision. Moreover, the phase shift of the static compacton can be observed. Before the interaction, the static compacton is located at $x = -1.5$ and after the interaction, it is shifted to $x = 1.5$. The phase shift of the static compacton is 3.

4.3. Interaction between semi-foldon and dromion

Finally, we discuss the interactive behavior between semi-foldon and dromion. Selecting

$$\chi_x = \text{sech}^2(\zeta) + 0.5\text{sech}^2(\zeta - 0.3t), \quad x = \zeta - 1.5\tanh(\zeta - 0.3t),$$

$$\chi = \int^{\zeta} \chi_x x_\zeta d\zeta, \tag{43}$$

$$\varphi = \tanh(y), \tag{44}$$

then the interaction between semi-foldon and dromion can be observed.

From Fig. 3, we can see the interaction between multi-valued semi-foldon and single-valued dromion is non-elastic which is different from two above cases. The amplitude of the static dromion is decreased a little, while the amplitude of the moving semi-foldon is increased. The shapes of the static dromion and the moving semi-foldon are both changed. Furthermore, the phase shift of the static dromion can be observed. Before the collision, the static dromion is located at $x = -1$ and after the collision, it is shifted to $x = 2$. The phase shift of the static dromion is also 3.

5. SUMMARY AND DISCUSSION

In summary, Using the extended projective Ricatti equation expansion method, abundant variable separation solutions of the $(2 + 1)$-dimensional dispersive long wave systems are obtained. From the special variable separation solution (38) and by selecting appropriate functions, new types of interaction between the multivalued and the single-valued solitons, such as semi-foldon and dromion, semi-foldon and peakon, semi-foldon and compacton are presented. There is a worthwhile question: why the field v expressed by (38) engenders rich excitations of the formula (1)? The main reason is that, compare formula (1) with the solutions (38), they are essentially equivalent. By selecting $p(x, t) = \exp[2\chi(x, t)]$, $q(y, t) = \exp[2\varphi(y)]$, $a_1 = a_2 = 0$, $a_0 = a_3 = 1$, one can derive $v = -\frac{2}{\lambda}U$. In a similar way, by choosing some apt parameters, one can also further verify the solution (33) and the formula (1) are essentially identical. Thus all the localized excitations based on the common formula (1) can be obtained from the solution (38).

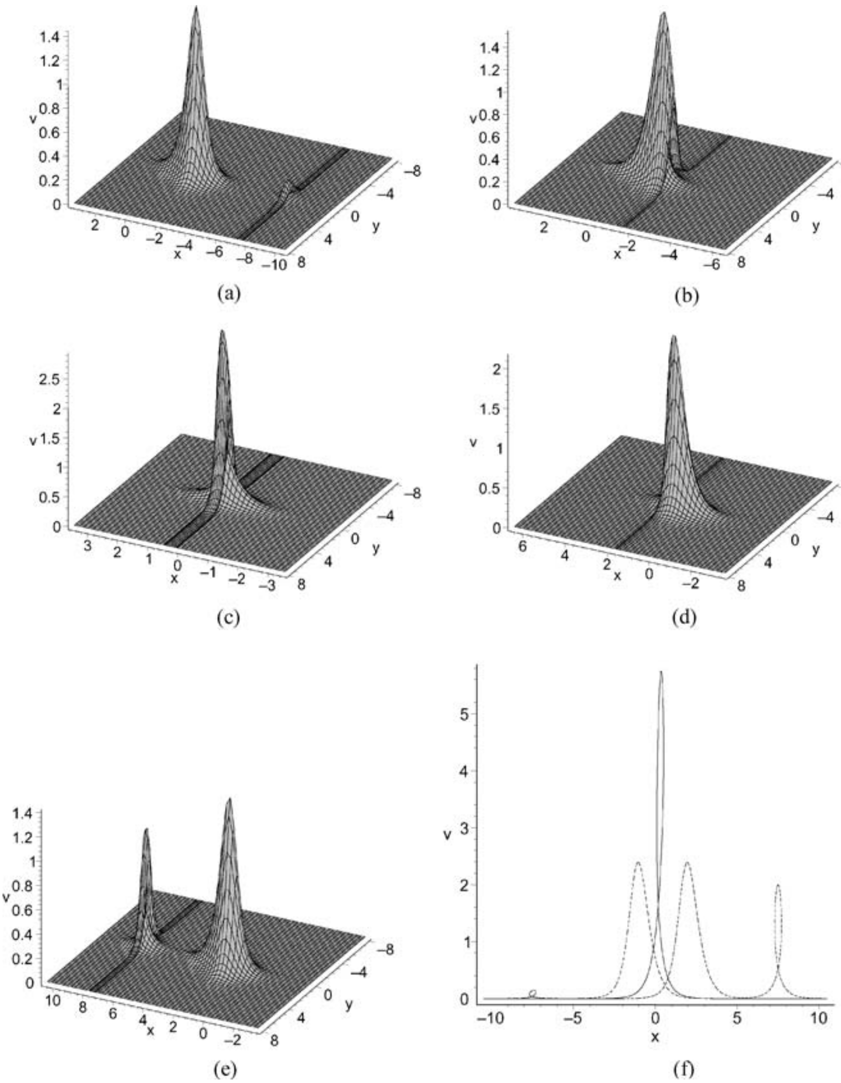

Fig. 3. Evolution profiles of the non-elastic interaction between semi-foldon and dromion determined by Eq. (38) with Eqs. (43) and (44) at (a) $t = -25$, (b) $t = -5$, (c) $t = 1$, (d) $t = 5$, (e) $t = 25$. (f) The corresponding sectional view at $\{t = -25, y = 0\}$ (dotted line before collision), $\{t = 1, y = 0\}$ (solid line in collision), $\{t = 25, y = 0\}$ (dashed line after collision), respectively.

This method presented in this paper is an initial work, more application to other nonlinear physical systems should be concerned and deserve further investigation. In our future work, on the one hand, we devote to generalizing this method to other $(2+1)$-dimensional nonlinear systems such as Broer-Kaup-Kupershmidt system, Boiti-Leon-Pempinelle system etc. On the other hand, we will look for more interesting localized excitations.

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